

The Roche School

CALCULATION

WORKSHOP



Aims of the Workshop

- To raise standards in calculating by working closely with parents.
- To provide parents with a clear outline of the key features of calculation teaching at The Roche School.
- To inform parents of our calculation strategies and the National Curriculum objectives

Mathematics Today

- **Mental calculation** skills are vital.
- Children need to have secure mental representations of numbers and number patterns. So number bonds to 20 (addition and subtraction, including bridging 10 should be known 'by heart') by KS2
- All times tables and their related divisions should (ideally) be known by the end of Spring Term in Year 4
- Children need the ability to **estimate**.

e.g. If I have 18 sweets in one bag and 33 sweets in another bag, how many do I have altogether?

Children can estimate by adding 20 and 30 and know that roughly the answer should be around 50.



National Curriculum Aims

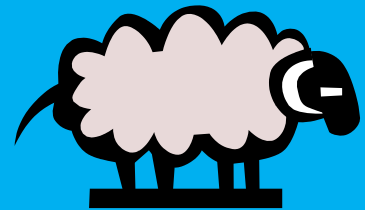
- **Fluency** in the fundamentals of mathematics
- **Reasoning** mathematically and **systematically**
- **Problem Solving**

Mathematics Today

- **Problem Solving:** Children need to learn maths in a **real life context**.

As well as knowing $7 \times 7 = 49$. Children need to be able to do the following:

There are 7 fields, each field has 7 sheep in them. How many sheep are there in total?



So, we approach calculating by giving a clearer understanding of 'what it looks like' in everyday life. This gives a stronger grounding for children into why they have to know it!

- Children need to be able to **explain** how they have calculated something using a secure method. If they can't explain it, they don't fully understand it.

Children need to understand the relationship between different calculations,

$$\text{e.g. } 2 + 3 = 5$$

$$\text{So, } 3 + 2 = 5$$

$$\text{So, } 5 - 3 = 2$$

$$\text{So, } 5 - 2 = 3$$

Each calculation can lead on to at least 3 other
THINGS YOU KNOW.

This understanding of maths as patterns and relationships really helps children as they progress onto more challenging problems and calculations (particularly long division).

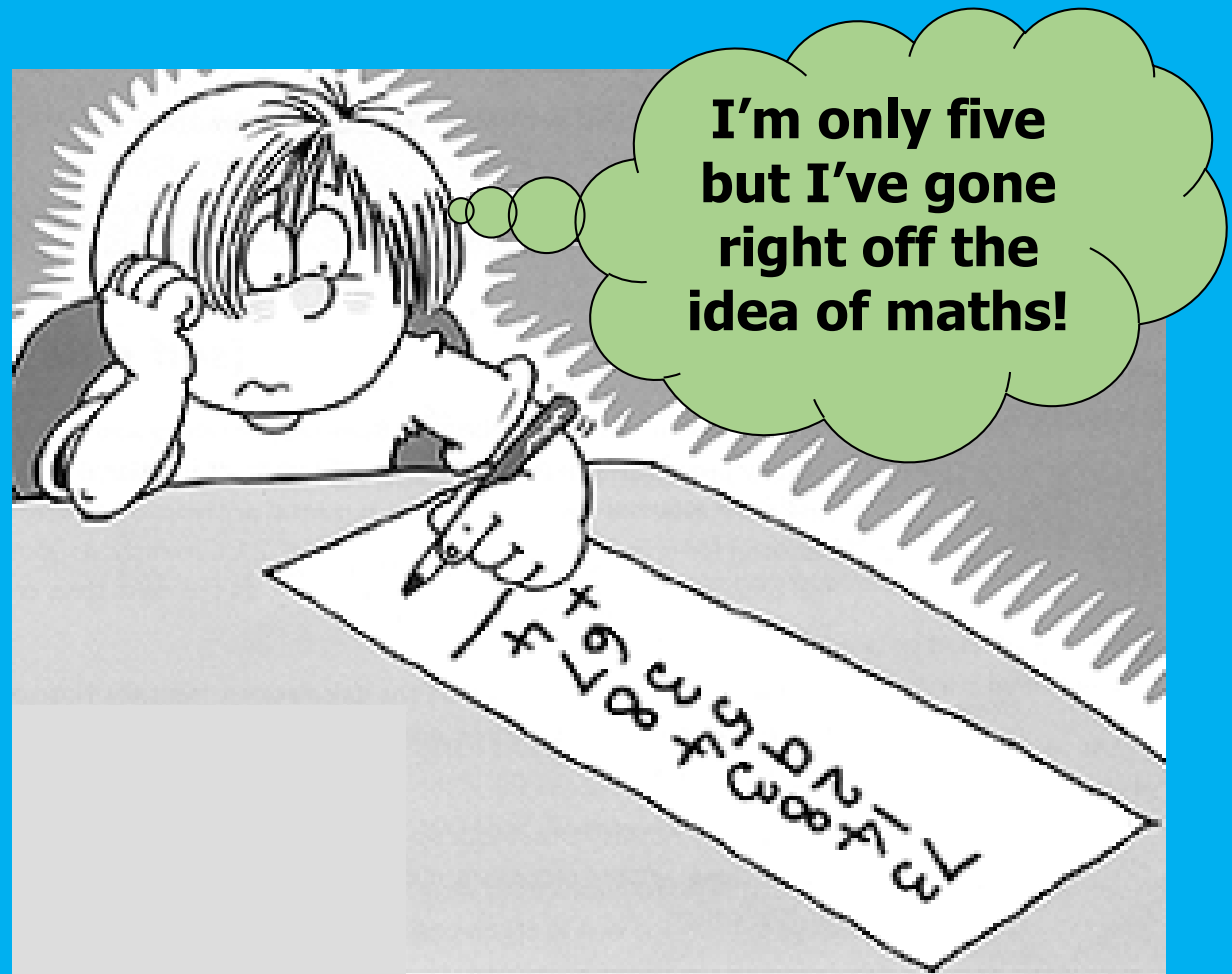
For today, I'll concentrate on what we would like a child to know in each year group – with particular emphasis on NUMBER WORK (CALCULATION) and methods.

Nationally we are now using a 'Mastery Approach', which enables us to stretch children who work to the expected standards quickly, by giving them tougher challenges on the calculation theme (for example more challenging problems to solve) being covered.

We are setting our expectations for calculation in line with National Standards, as I believe they are challenging, but achievable. We will, of course, give extra guidance and support to children who find the expectations too high. (e.g. times tables, by what age, etc)

Mental before written

- We need to first develop a sense of number.
- *When the children are secure in their understanding, written calculation can be taught quickly, with fewer errors made.*
- They can also grasp more complex topics taught in KS2.



quicker

UNDERSTANDING CALCULATION

From CONCRETE, through PICTORIAL to ABSTRACT

CONCRETE

PICTORIAL

ABSTRACT

Arrays-
showing
commutative
multiplication

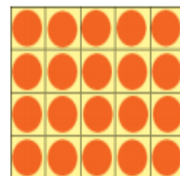
Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.

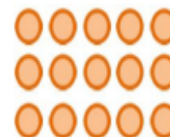
$$\begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array} \quad \begin{array}{l} 4 \times 2 = 8 \\ 2 \times 4 = 8 \end{array}$$

$$\begin{array}{c} \bullet \bullet \\ \bullet \bullet \\ \bullet \bullet \\ \bullet \bullet \end{array} \quad \begin{array}{l} 2 \times 4 = 8 \\ 4 \times 2 = 8 \end{array}$$



Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



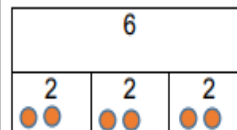
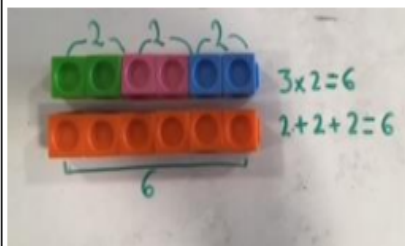
$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

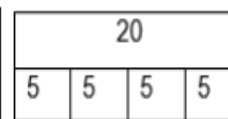
$$3 \times 5 = 15$$

Bar models
representing
multiplication.



$$2 + 2 + 2 = 6$$

$$3 \times 2 = 6$$



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$3 \times 2 = 6$$

$$4 \times 5 = 20$$

The Importance of Concrete Resources

- Concrete resources allow children to take an abstract concept and develop a visual/mental concept. They are objects you can physically manipulate to help you to grasp concepts.
- The more time spent developing these core concepts, the children to improve their confidence and accuracy.
- This stage should not be skipped, even for the most able children.
- Visual and kinaesthetic learning uses different areas of the brain- building links and strengthening understanding.



Even in Year 6, concrete resources are used to allow the children to develop their understanding of new concepts such as ratio or algebra.

Examples of concrete resources

- Numicon (used particularly to visualise number bonds)
- Unifix (useful all the way up to understanding ratio in Years 5 and 6)
- Deines blocks (good for recognising place value in base 10)
- Clocks
- Rulers and metre sticks
- FINGERS!



Everyday objects (example from Year 4: to explain $8 \times 3 \times 6$, we used a pencil pot with 8 compartments and put 3 pencils in each compartment. How many pencils were there? Then we imagined having having 6 pencil pots. How many pencils did we have then?)

Numicon



- Number ideas are very abstract, and children need to have these presented to them in a wide variety of ways
- **Numicon** is multi-sensory approach using patterned shapes, rods and number lines
- **Numicon** patterns themselves physically fit together so that when combining numbers children can physically 'do' the calculations that we want them to 'think'
- This in turn helps them to internalise these concepts
- **Numicon** helps to build confidence and fluency

Examples of pictorial resources

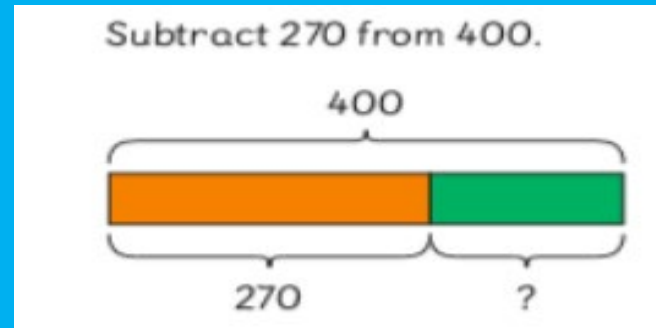
These are any visual aids that 'show' what is either being represented by an abstract mathematical 'number sentence', or an aid to calculating. They can be:

Number lines

Bar models

Pictorial jotting

Diagrams

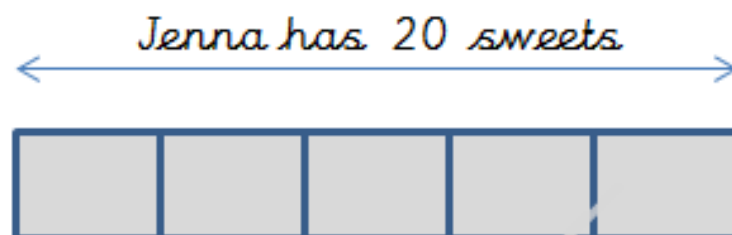


LET'S USE IT! (LAPs/Aps)

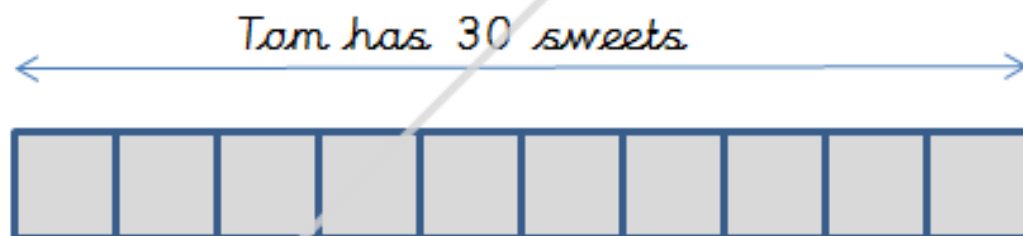
Jenna has 20 sweets and shares them between 5 friends.
Tom has 30 sweets and shares them between 10 friends.

Whose friends will get the most sweets? _____

Can you use bar models to work this out



$$20 \div 5 = \boxed{}$$



$$30 \div 10 = \boxed{}$$

The stages to written calculation fluency

IMPORTANT! Being fluent in calculating (being able to DO all the established written methods) is only worthwhile if you know WHEN to apply them.

We will show you the steps to formal written methods, via extended methods that help to understand the concepts involved. Sometimes these 'extended' written methods may create additional issues and so we recognise that sometimes 'moving on' to the shorter versions can be helpful once the over-riding understanding of the concept is reached.

A NEW NATIONAL CURRICULUM FOR MATHEMATICS SINCE AUTUMN 2014

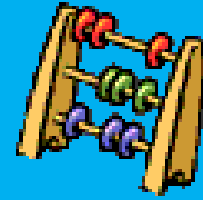
- By the time the children are at the end of year 3 they must be able to use a formal written method for all operations and the differentiated outcomes throughout the years will be the size of the numbers the children are using. Due to this ... we will all be working towards using the same methods to calculate in the 4 operations BY VARIOUS POINTS IN THE UPPER SCHOOL.
- Addition: column method
- Subtraction: column method
- Multiplication: short multiplication moving to long multiplication
- Division: short division moving to long division

Please note: These are the 'Official' formal methods the school will teach – if your child has learnt a similar, or different method that is successful for them, they are allowed to continue to use it.

Number: Addition and Subtraction

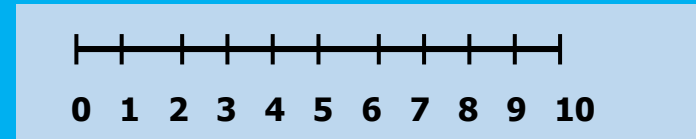
WRITTEN METHODS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Mental Calculation)		add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	
INVERSE OPERATIONS, ESTIMATING AND CHECKING ANSWERS					
	<u>recognise</u> and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	estimate the answer to a calculation and use inverse operations to check answers	estimate and use inverse operations to check answers to a calculation	use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	<u>use</u> estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Addition



1. Practical addition of real objects.
2. Mental addition of number facts (e.g. Number bonds).

3. Use of a structured number line to add.



4. Partitioning to add.

$$\boxed{1} \boxed{23} = \boxed{100} + \boxed{20} + \boxed{3}$$

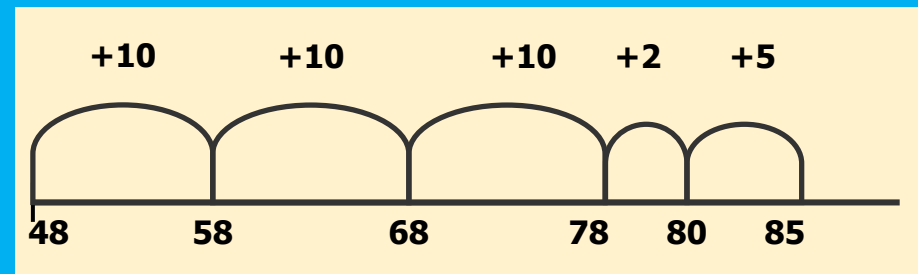
5. Use of an unstructured number line.

$$37 + 48 =$$

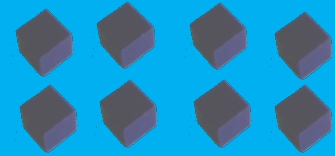
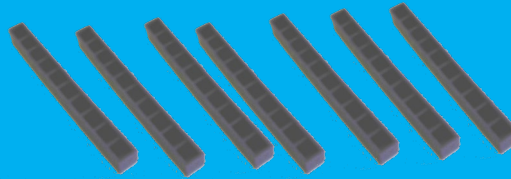
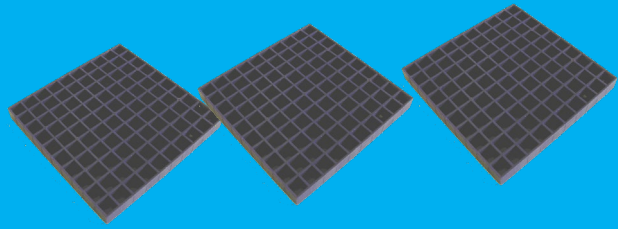


Remember to
put the largest
number first

Note: the units jump can be
broken down to make it easier
to count on through a multiple
of 10



Partitioning



$$378 = 300 + 70 + 8$$

$$\text{Or } 378 = 300 + 60 + 18$$

$$\text{Or } 378 = 200 + 170 + 8$$

Addition

6. Beginning to record vertically.

Adding the least significant digit first.

This is usually introduced in Year 3, once the children have secure mental strategies.

$$126 + 57 =$$

Estimate: $126 + 57$ is nearly $130 + 60$ so estimate answer should be near 190.

$$\begin{array}{r} 126 \\ + 57 \\ \hline 13 \quad (6+7) \\ 70 \quad (20+50) \\ 100 \quad (100+0) \\ \hline 183 \end{array}$$

Addition

7. Standard vertical method involving carrying.

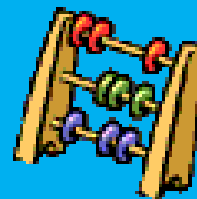
When children are confident working with larger numbers using the previous strategies, they will be introduced to 'carrying' digits. Usually this is either in Year 3 or Year 4 and SHOULD be in place by Year 5. **2856+1095**

Estimate: 2900+1100 =4000 Answer should be less as I have rounded up.

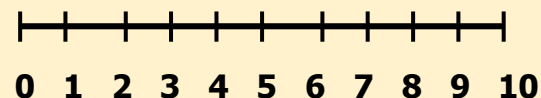
$$\begin{array}{r} 2856 \\ + 1095 \\ \hline 3951 \\ \hline 11 \end{array}$$

Subtraction

1. Practical subtraction of real objects.
2. Mental subtraction of number facts.
3. Use of a structured number line to add.

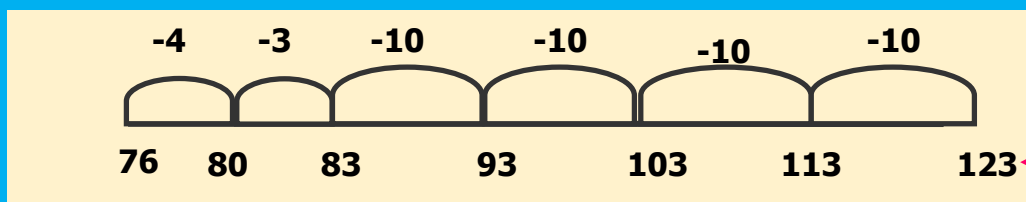


4. Use of an unstructured number line.
 $123 - 47 =$



Estimate first $120 - 50 = 70$

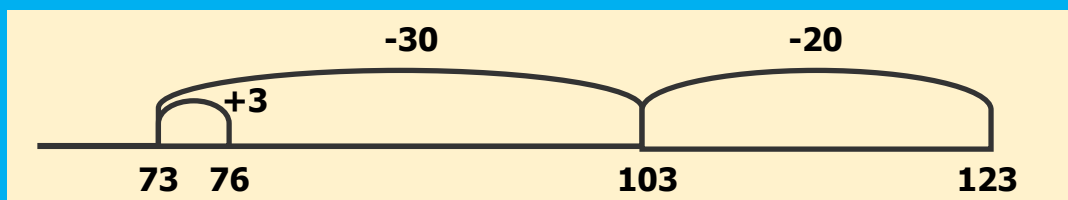
Counting back- (most significant digit first, in this case tens, then units)



Start here.



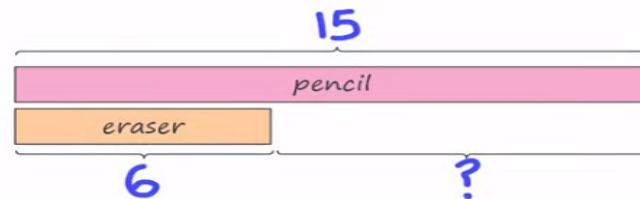
or



Subtraction

5. Counting on (Complimentary addition)

A pencil has a length of 15 cm.
An eraser has a length of 6 cm.
How much longer is the pencil than the eraser?

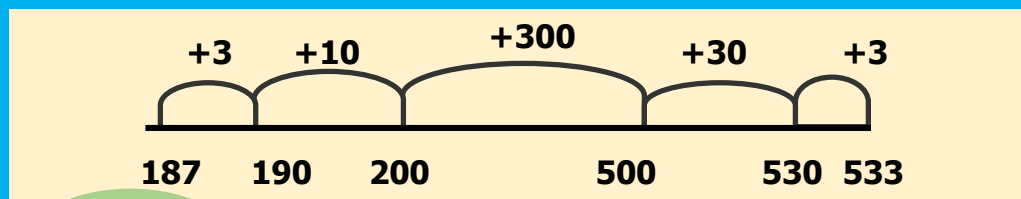


$$15 - 6 = 9$$

Children will be taught to find the difference by counting on in the following ways.

$$533 - 187 =$$

Estimate : $530 - 190 = 340$ (carried out mentally as $530 - 200 + 10$
 $= 340$)



Start at
this
end.

Add the numbers
on top of the
number line to
get the answer.



Subtraction

7. Subtraction by decomposition

Children will then be shown decomposition; they must really understand place value to do this.

$$\begin{array}{r} 83 \\ - 55 \\ \hline \end{array} \text{ is the same as } \begin{array}{r} 80 + 3 \\ 50 + 5 \\ \hline \end{array}$$

Ten is taken from 80 and added to the three.



This can be rewritten as

$$\begin{array}{r} 70 + 13 \\ - 50 + 5 \\ \hline 20 + 8 = 28 \end{array}$$

A hundred is taken from 500 and added to 20.

$$\begin{array}{r} 533 \\ - 187 \\ \hline \end{array} \text{ is the same as } \begin{array}{r} 500 + 30 + 3 \\ - 100 + 80 + 7 \\ \hline \end{array} \begin{array}{r} 500 + 20 + 13 \\ - 100 + 80 + 7 \\ \hline \end{array}$$

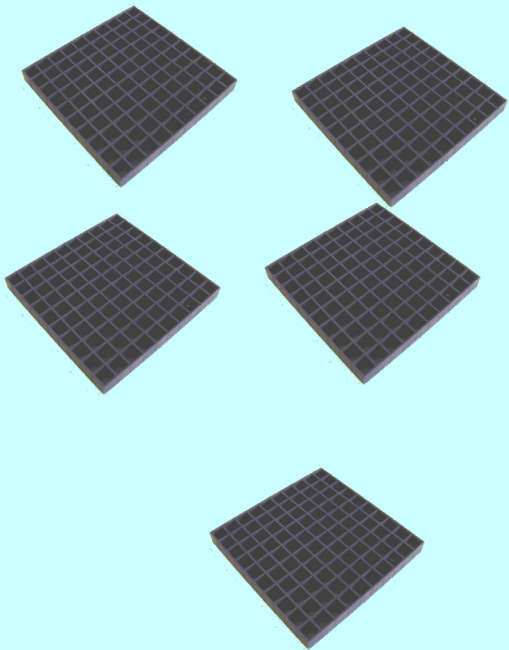
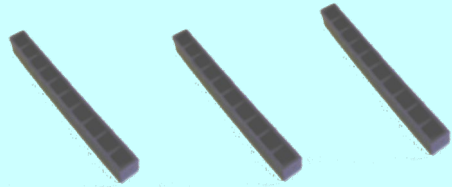
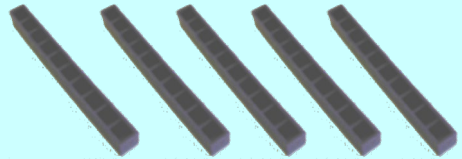
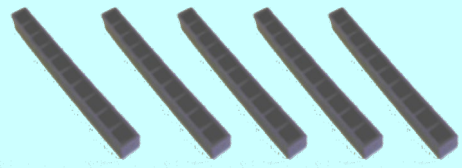
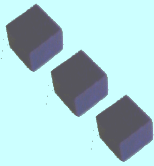
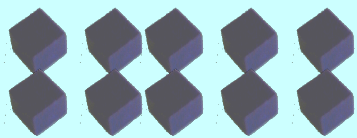
A hundred now needs to be moved as well.

$$\begin{array}{r} 400 + 120 + 13 \\ - 100 + 80 + 7 \\ \hline 300 + 40 + 6 = 346 \end{array}$$



Subtraction

$$\begin{array}{r} 533 \\ -187 \\ \hline \end{array} = \begin{array}{r} 500 + 30 + 3 \\ 100 + 80 + 7 \end{array} = \begin{array}{r} 500 + 20 + 13 \\ 100 + 80 + 7 \end{array} = \begin{array}{r} 400 + 120 + 13 \\ 100 + 80 + 7 \end{array} = \mathbf{346}$$

H	T	U
	  	 

Subtraction

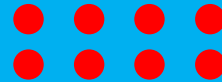
8. Standard vertical subtraction with transferring

Only when children are completely secure in this we will teach them standard vertical subtraction with transferring (ideally by the end of Year 3 into Year 4).

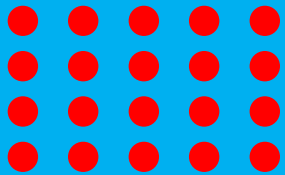
$$\begin{array}{r} \\ \cancel{5} \cancel{3} \\ - \\ \hline \end{array}$$

Multiplication

1. Practical Multiplication - 2×4 2 lots of 4.



2. Use of arrays 4×5



This is
an
array.



3. Repeated addition

$$4 \times 5 =$$

$$5 + 5 + 5 + 5 = 20$$

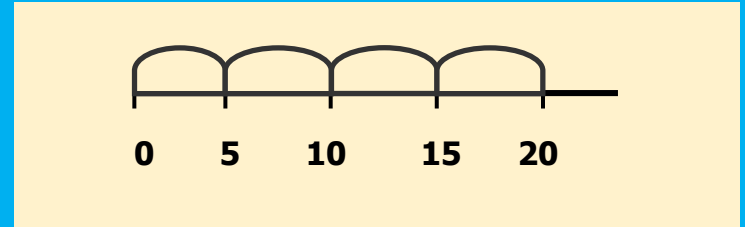
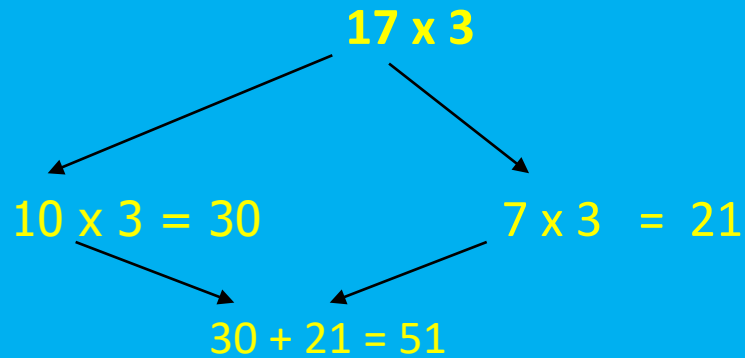
$$\text{or } 4 + 4 + 4 + 4 + 4 = 20$$

Multiplication

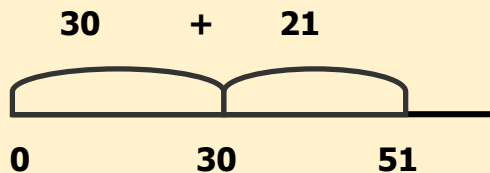
4. Repeated addition can also be done on a number line.

$$4 \times 5$$

5. Partitioning – Simple recording



Number lines
can be used
to do the
addition part!



Multiplication

4. The Grid Method This is our strategy for beginning to formally record multiplication. $17 \times 3 = (10 \times 3) + (7 \times 3)$

X	10	7
3	30	21

$$30 + 21 = 51$$

Add the numbers inside the grid together to get the answer.



5. Multiplying two 2 digit numbers 18×23
Estimate $20 \times 20 = 400$.

?

X	10	8
20	200	160
3	30	24

Try to add the numbers together mentally. If not, use a written method.

$$200 + 160 + 30 + 24 = 360 + 54$$
$$360 + 54 = 414$$

$$\begin{array}{r} 360 \\ + 54 \\ \hline 4 \\ 110 \\ 300 \\ \hline 414 \end{array}$$

Multiplication

8. Once children are confident with the grid method (OR if they find it too cumbersome and are making errors despite understanding the concept), they will be introduced to the following strategies for recording.

Short multiplication

$$17 \times 3$$

$$\begin{array}{r} 17 \\ \times 3 \\ \hline 21 \quad (7 \times 3) \\ 30 \quad (10 \times 3) \\ \hline 51 \end{array}$$

leads to

$$\begin{array}{r} 17 \\ \times 3 \\ \hline 51 \\ 2 \end{array}$$

Which then leads on to HTU X U in Year 4

9. Long multiplication is introduced either later in Year 4, or usually Year 5

e.g. 184×32

Estimate $180 \times 30 = 5400$.

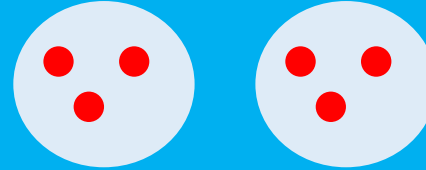
$$\begin{array}{r} 184 \\ \times 32 \\ \hline 368 \quad (184 \times 2) \\ 5520 \quad (184 \times 30) \\ \hline 5888 \end{array}$$

Division

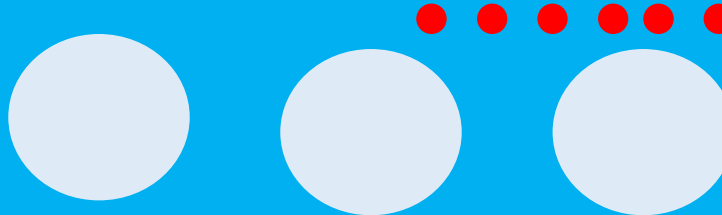
1. Sharing or Grouping – Division is initially represented pictorially.

$$6 \div 2 = 3$$

6 sweets shared between 2 people. How many each?

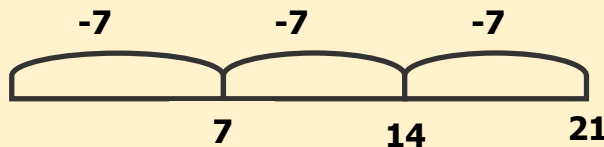


There are 6 people in a room. Put them into groups of 2. How many groups can you make?



Sharing and grouping are two totally different concepts that children need to understand.

2. Using a number line to show division.



$$21 \div 7 = 3$$

Division

3. Chunking- Using Multiples of the Divisor

$$90 \div 5 = 18$$

$$\begin{array}{r} 90 \\ - 50 \quad (10 \times 5) \\ \hline 40 \\ - 40 \quad (8 \times 5) \\ \hline 0 \end{array}$$

Start with 90
and take away
multiples of 5.



4. Using Chunking with remainders

$$87 \div 4 = 21 \text{ r } 3$$

$$\begin{array}{r} 4 \overline{) 87} \\ - 40 \quad (10 \times 4) \\ \hline 47 \\ - 40 \quad (10 \times 4) \\ \hline 7 \\ - 4 \quad (1 \times 4) \\ \hline 3 \end{array}$$

Division

5. Using Chunking with larger numbers.

$$875 \div 24 = 36 \text{ r } 11$$

$$\begin{array}{r} 24 \overline{) 875} \\ - 240 \quad (10 \times 24) \\ \hline 515 \\ - 480 \quad (20 \times 24) \\ \hline 395 \\ - 360 \quad (15 \times 24) \\ \hline 155 \\ - 120 \quad (5 \times 24) \\ \hline 35 \\ - 24 \quad (1 \times 24) \\ \hline 11 \end{array}$$

Division

6. Short Division either later in Year 4 (if competent with understanding – NOTE: this is earlier than NATIONAL Curriculum guidelines, but can help) or Year 5

$$875 \div 5 = 175$$

$$\begin{array}{r} 175 \\ 5 \overline{) 875} \end{array}$$

7. Long Division in Year 5 or Year 6

With this method, we would expect to see some additional calculating 'outside of the main calculation, using estimation to support.

$$\begin{array}{r} 1813 \\ 13 \overline{) 23576} \\ \underline{13} \\ 105 \\ \underline{104} \\ 17 \\ \underline{13} \\ 46 \\ \underline{39} \\ 7 \end{array}$$

13
26
39
52
65
78
91
104

- An important thing to remember when developing a child's mathematical skills is LITTLE AND OFTEN!
10 minutes each day working on a method, or number bonds, or times tables, OR TELLING THE TIME, will reap rewards much more quickly than spending a long time 'NUMBER CRUNCHING' once a week.

Secondly, when 'doing' maths with a child, try to give 'real-life' problems where number is used (e.g. using money). Understanding word problems will also help with their reading and comprehension.

Also, as your child becomes more competent, give problems that require more than one step to solve, and include more than one calculating function.

Review of Core Principles

- A focus on mental calculations.
- The ability to estimate.
- To use maths in a real life context.
- To ask children to explain how they have calculated something using a method that suits them.
- Teach children written calculations, but only when children are ready.

MOST IMPORTANTLY:

PLEASE HAVE FUN WITH YOUR BUDDING MATHEMATICIANS!

USEFUL LINKS:

NCETM: <https://www.ncetm.org.uk/>
<https://www.ncetm.org.uk/resources/48070>

Calculation guidance

<https://www.ncetm.org.uk/public/files/25120980/NCETM+Calculation+Guidance+October+2015.pdf>